STOCHASTIC MODELLING OF PARTICLE SEGREGATION IN A HORIZONTAL DRUM MIXER*

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A process of segregation of two distinct fractions of solid particles in a rotating horizontal drum mixer was described by stochastic model assuming the segregation to be a diffusion process with varying diffusion coefficient. The model is based on description of motion of particles inside the mixer by means of a stochastic differential equation. Results of stochastic modelling were compared to the solution of the corresponding Kolmogorov equation and to results of earlier carried out experiments.

In our previous paper[1] we have made an attempt to describe the process of axial segregation of two fractions of solid particles, differing in size, inside a horizontal drum mixer by means of a one-dimensional diffusion equation

$$\frac{\partial}{\partial \theta} x(z,\theta) + \frac{\partial}{\partial z} [V(z,\theta)\, x(z,\theta)] - \frac{\partial^2}{\partial z^2} [D(z,\theta)\, x(z,\theta)] = 0 , \qquad (1)$$

where θ is the time and z the axial coordinate. Function $x(z, \theta)$ denotes the concentration of larger particles at position x at time θ. $V(z, \theta)$ is a drift velocity and $D(z, \theta)$ diffusion coefficient. We supposed D to be a known function of axial coordinate and, as the case may be, also of time. A form of the function $D(z, \theta)$ must express the fact that in the proximity of side walls of the mixer the segregation of distinct particle fractions takes place. An explanation of this phenomenon was given in a satisfactory way by Donald and Roseman[2,3]. If the particles in the mixer are of the same size, shape and density the segregation does not take place. However, Eq. (*1*) is still valid and can be used for description of mixing of solid particles differing, for example, only in a

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colour. An assumption that D is of constant magnitude with respect to position and time is fully legitimate in such a case[4,5].

In the above cited paper[1] it was proved that the drift velocity $V(z, \theta)$ may be neglected. This approach is, from the point of view of diffusion processes theory, more correct than assumption of non-zero value of $V(z, \theta)$ as was used for description of that type of processes by Fan and Shin[6].

We solved the differential equation (*1*) applying boundary conditions

$$\frac{\partial}{\partial z}[D(z,\theta)\, x(z,\theta)] = 0\,; \quad (z = \pm L')\,, \tag{2}$$

where L' is a half-length of the mixer, and initial condition

$$x(z, 0) = x_0(z)\,, \tag{3}$$

i.e. for prescribed initial distribution $x_0(z)$ of greater particles along axial coordinate of the mixer.

The solution of Eq. (*1*) was compared to data obtained in experiments carried out earlier[7]. In the case of so called "pure segregation", i.e. for uniform initial distribution ($x_0(z)$ = const.), it was shown that the proposed model successfully fits experimental data. In the case of other than uniform initial distribution more sophisticated model comprising both processes of particle mixing and of segregation had to be used. However, despite of higher model complexity less precise fitting of experimental data was achieved.

Further on we have proved[1] that in the case of "pure segregation" the diffusion coefficient is a function only of axial coordinate, not of time, its value being indirectly proportional to the value of stationary concentration at given position

$$D(z) = \lim_{\theta \to \infty} B/x(z,\theta) = B/x_\infty(z)\,. \tag{4}$$

Values of function $x_\infty(z)$ were evaluated by interpolation of averaged experimental values of stationary concentration of larger particles.

THEORETICAL

In the theory of diffusion processes[8,9] it is demonstrated that following Kolmogorov forward diffusion equation is equivalent to Eq. (*1*)

$$\frac{\partial f}{\partial \theta} + \frac{\partial}{\partial z}[V(z,\theta) f] - \frac{\partial^2}{\partial z^2}[D(z,\theta) f] = 0\,. \tag{5}$$

So called transitive probability density f is a solution of Eq. (*5*)

$$f = f(z;\theta \mid z_0;\theta_0) = \lim_{z \to 0} \frac{1}{\Delta z} P\{z \le Z(\theta) < z + \Delta z \mid z_0 = Z(\theta_0)\} \quad (\theta > \theta_0) \tag{6}$$

The probability density f describes e.g. one-dimensional random motion of a material particle, the position of which $Z(\theta)$ is a stochastic function of time defined at each θ by the inequality in Eq. (*6*), under the condition that in certain preceding time θ_0 the particle was located at a position z_0 (see footnote)*.

It was proved[8,9], that the stochastic process $Z(\theta)$ may be also described by the stochastic differential equation

$$dZ(\theta) = V[Z(\theta), \theta]\, d\theta + [2D(Z(\theta), \theta)]^{1/2}\, dW(\theta)\,, \qquad (7)$$

where the functions V and D are identical with that in Eq. (*5*). The function $W(\theta)$ is a source of randomness of the process $Z(\theta)$. It is usually called Wiener process and is defined as the stochastic process with Gaussian distribution, zero mean value and variance equalling length of time interval elapsed from the origin of the process. Process $W(\theta)$ itself has zero value at the origin. A solution of Eq. (*7*) satisfies to transitive probability density f defined by Eq. (*6*).

Both approaches, i.e. solution of Kolmogorov equation and direct numerical solution of stochastic differential equation, will be used for description of one-dimensional diffusion of solid particles in a horizontal drum mixer with length L ($L = 2L'$) – see Fig. 1. We shall assume symmetry of the diffusion (mixing) process with respect to the centre of the mixer identical with the origin of axial coordinate z. We shall further consider uniform initial distribution of particle fraction under consideration inside of the mixer

$$f_0(z,\theta_0) = \begin{cases} 1/L\,, & [-L' \le z \le +L'] \\ 0\,, & [|z| > L'] \end{cases} \qquad (\theta_0 = 0) \qquad (8)$$

As it was stated above we shall assume zero value of the drift velocity and the case of so called "pure segregation" will be considered, i.e. diffusion coefficient will be function only of particle position on the axial coordinate.

The conditions of elastic reflection of particles on the side walls of the mixer may be expressed by relation (cf. Eq. (*2*))

$$\frac{\partial}{\partial z}[D(z) f(z;\theta)] = 0\,, \qquad (z = \pm L')\,. \qquad (9)$$

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* By this way we define the particle as a material point, in the case of the particle having defined size it is necessary to consider $Z(\theta)$ as the coordinate of the particle centre of gravity.

Under fulfilment of the above conditions following equation for stationary probability density f is valid

$$D(z) \lim_{\theta \to \infty} f(z;\theta) = D(z) f_\infty(z) = b = \text{const.} \qquad (10)$$

After introducing dimensionless coordinates of time and particle position

$$y = z/L\ , \quad t = \theta b/L\ , \qquad (11)$$

we can define dimensionless transitive probability density and dimensionless diffusion coefficient H

$$\varphi(y;t) = L f(yL; tL/b)\ , \ H(y) = D(yL)/bL = 1/\varphi_\infty(y)\ . \qquad (12)$$

Then Eq. (5) may be rewritten in dimensionless form

$$\frac{\partial}{\partial t}[\varphi(y;t)] = \frac{\partial^2}{\partial y^2}[\varphi(y;t)/\varphi_\infty(y)]\ , \qquad (13)$$

with initial condition

$$\varphi(y;0) = \varphi_0 = \begin{cases} 1\ , & [-1/2 \leq y \leq +1/2] \\ 0\ , & |y| > 1/2 \end{cases} \qquad (14)$$

and boundary conditions

$$\frac{\partial}{\partial y}[\varphi(y;t)/\varphi_\infty(y)] = 0\ , \ (y = \pm 1/2)\ . \qquad (15)$$

Using the same assumptions as in the three previous paragraphs we can write corresponding stochastic differential equation[8,9], expressed now in an integral form

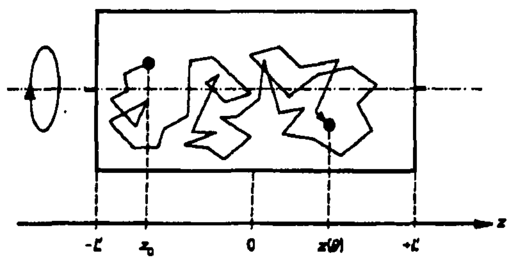

FIG. 1
Random motion of one of greater particles inside of rotating drum mixer

$$Z(\theta) = Z(0) + \sqrt{2}\int_0^{\theta} [D(Z(\tau))]^{1/2}\, dW(\tau)\,. \tag{16}$$

In a way analogous to Eqs (*11*) and (*12*) we can define dimensionless variables

$$Y(t) = Z(tL/b)\,, \;\; H[Y(t)] = D[Z(tL/b)/L]/bL\,. \tag{17}$$

A definition of "dimensionless" Wiener process, with respect to numerical solution of the stochastic differential equation and possibility comparing results with the solution of diffusion equation (*13*), requires some additional considerations: According to Ito's definition of stochastic integral Eq. (*16*) may be written in the form

$$[Z(\theta) - Z(0)]/\sqrt{2} = \lim_{\lambda\to 0}\sum_{i=1}^{n} [D(Z(\theta_{i-1}))]^{1/2}\,[W(\theta_i) - W(\theta_{i-1})]\,, \tag{18}$$

where $0 = \theta_0 < \theta_1 < ... \theta_{i-1} < \theta_i < ... \theta_n = \theta$, $\lambda = \max_i(\theta_i - \theta_{i-1})$ and $D(Z(\theta))$ is (with probability equalling one) a continuous function of time. Distribution function of Wiener process is according to its definition given by relation

$$P\{W(t) < w\} = \int_{-\infty}^{w} \exp(-u^2/2t)/(2\pi t)^{1/2}\, du\,. \tag{19}$$

Let us denote

$$\alpha = \theta_i - \theta_{i-1}\,, \tag{20}$$

and consider α to be constant for all values of i. According to Feller[10] we shall introduce an identity operator $X_i(\theta) \stackrel{d}{=} X_j(\theta)$ for all stochastic processes having identical distribution function. Considering that Wiener process $W(t)$ has Gaussian distribution function and, consequently, independent increments, we can write equation

$$W(\theta_i) - W(\theta_{i-1}) \stackrel{d}{=} W(\alpha) \stackrel{d}{=} \sqrt{\alpha}\, G_i\,, \tag{21}$$

where G_i is a random variable with Gaussian distribution $N(0,1)$. Then Eq. (*18*) may be replaced by

$$[Z(\theta) - Z(0)]/\sqrt{2} \stackrel{d}{=} \lim_{\alpha\to 0}\sqrt{\alpha}\sum_{i=1}^{n} [D(Z(\theta_{i-1}))]^{1/2}\, G_i\,. \tag{22}$$

Let us further define dimensionless time increment

$$\gamma = \alpha b/L\,. \tag{23}$$

Dividing Eq. (*22*) by the mixer length L and considering Eq. (*17*) we can finally write "dimensionless" stochastic differential equation

$$Y(t) \stackrel{d}{=} Y(0) + \sqrt{2}\lim_{\gamma\to 0}\sqrt{\gamma}\sum_{i=1}^{n} [H(Y(t_{i-1}))]^{1/2}\, G_i =$$

$$= Y(0) + \sqrt{2} \int_0^{\theta} [H(Y(s))]^{1/2}\, dU(s)\,. \qquad (24)$$

The stochastic function of time $U(s)$ defined in Eq. *(24)* may be called "dimensionless" Wiener process. Its increments can be approximated by relation

$$\Delta U(t) = \sqrt{\gamma}\, G\,. \qquad (25)$$

The initial value of stochastic process $Y(t)$ has a distribution defined by Eq. *(14)*.

Now we shall consider the elastic reflection of particles at the ends of the interval $< -0.5, +0.5 >$, (i.e. on the side walls of the mixer) in such a sense: If in the progress of numerical simulation using fixed value of time increment $\gamma > 0$ at any time $t = \gamma n$ a value of $Y(t)$ will overlap boundaries of that interval, then the reflection of particle back into the interval will be considered. The length of particle trajectory after the reflection equals the length of overlap, but the particle moves in opposite direction.

The solution of Eq. *(24)*, i.e. the stochastic function of time $Y(t)$ is described by following distribution function

$$\Phi(y) = P\left\{-1/2 \le Y(t) < y\right\}, \qquad (\Phi(1/2) = 1)\,, \qquad (26)$$

the first derivative of which

$$\varphi(y) = d\Phi(y)/dy \qquad (27)$$

is solution of Eq. *(13)*. It enables us to compare the results of segregation modelling applying Kolmogorov equation to results of numerical simulation based on application of the stochastic differential equation.

EXPERIMENTAL

Experimental data presented in this paper were measured by Rochowiecki[7] in a model mixer (length 12 cm, diameter 7.4 cm) filled with two fractions of a sea sand (particle size 0.385 – 0.43 mm and 0.2 – 0.25 mm). Concentration of greater particles was measured at 11 sampling ports evenly located along the mixer. Details are given in paper of Rochowiecki.

Numerical Computations

Numerical computations reported in following paragraphs were executed on a personal computer AT/286 with 640 kB RAM and 80287 mathematical co-processor. All programmes were coded in Turbo PASCAL (Borland, ver. 5.5) using extended range arithmetics.

Interpolation of stationary concentration profile. The values of diffusion coefficients D and H can be easily calculated using Eqs *(10)* or *(12)* if value of stationary probability density $f_\infty(z)$ or $\varphi_\infty(y)$ is known at any point of mixer axial coordinate. Probability density $f_\infty(z)$ is related to particle concentration $x_\infty(z)$ by equation

$$f_\infty(z) = x_\infty(z) \Big/ \int_{-0.5}^{0.5} x_\infty(u)\, du\,. \qquad (28)$$

However, as it was mentioned above, there were only 11 measured values of stationary concentration along the mixer at our disposal. Therefore, we have used cubic spline interpolation[11] for evaluation of stationary concentrations at arbitrary position between experimental (grid) points. The values of stationary concentration at the boundaries of mixer (on side walls) were not ascertainable by measurement[1,7]. We have estimated them by trial-and-error method until condition

$$\int_{-0.5}^{0.5} \hat{\varphi}_{\infty}(y)\, dy = 1 \tag{29}$$

was fulfilled, where $\hat{\varphi}_{\infty}(y)$ is cubic spline approximation of stationary probability density.

A value of constant b in Eq. (*10*) was determined by simple regression method described earlier[1] using concentration profiles measured at 2, 4, 6 and 10 min from beginning of the process. The value of b was $8.63 \,.\, 10^{-6}$ m s^{-1}.

Numerical solution of Kolmogorov equation. Kolmogorov equation (*13*) with initial condition (*14*) and boundary conditions (*15*) was solved using finite difference method (Crank–Nicolson scheme)[12] with dimensionless axial coordinate step $h = 0.005$ and dimensionless time step $k = 0.0001$. The absolute error of dimensionless probability density φ was less than $1 \,.\, 10^{-5}$.

Numerical solution of stochastic differential equation. Random numbers G were generated by method of inverse distribution function interpolation[13]

$$G = F^{-1}(M) \ , \tag{30}$$

where F^{-1} is inverse Gaussian distribution function and M is uniform random variable on interval $< 0, 1 >$. Standard procedure RANDOM of Turbo PASCAL was used for generation of M values. The algorithm of Buttler[13] was adopted for interpolation of F^{-1} in Eq. (*30*). The grid values of F^{-1} were evaluated by numerical integration of Gaussian probability density function. 2 049 grid points were used for interpolation.

The stochastic differential equation (*24*) (SDE) was solved for a set of $N_p + 1$ hypothetical modelling particles distributed at origin ($t = 0$) evenly along the mixer axis, i.e. on the interval $< -0.5, +0.5 >$.

$$Y_{0,j} = -0.5 + j/N_p \,, \quad j = 0, 1, 2, \ldots, N_p \tag{31}$$

(cf. initial condition (*14*)), where $Y_{0,j}$ denotes the position of j-th particle at time $t = 0$.

Euler's method[12] with constant time step k was used for solution of Eq. (*24*) for each particle. SDE was rewritten to form

$$Y_{n+1,j} = Y_{n,j} + \Delta Y_{n,j} \,, \quad n = 0, 1, 2, \ldots \tag{32}$$

where

$$\Delta Y_{n,j} = \{2k/[\hat{\varphi}_{\infty}(Y_{n,j})]\}^{1/2}\, G_j \tag{33}$$

is an increment of j-th particle position in n-th step of solution. Initial positions of particles are given by Eq. (*31*).

According to Eqs (*32*) and (*33*) positions of all $N_p + 1$ particles were computed consecutively for $n = 0$, 1, 2, ...; stationary state was reached at $n \approx 4\,000$ using time step $k = 0.0001$. The positions of the particles $Y_{n+1,j}$ were checked at each step of numerical simulation and the particle reflection was considered if following conditions was fulfilled

$$\left| Y_{n,j} + \Delta Y_{n,j} \right| > 0.5 \,. \tag{34}$$

The length of mixer was divided to N_i intervals (classes) of identical width ΔY and relative frequencies of particles occurrence in each class were computed at each m-th step of solution ($m \geq 1$)

$$p_i = n_i/(N_p + 1)\,, \tag{35}$$

where n_i is a number of particles in i-th class, i.e. particles with position in the interval $< i\,\Delta y,\ (i+1)\,\Delta y >$. The relative frequencies p_i were compared to Kolmogorov equation solution using following relation that converts probability density $\varphi(y;t)$ to ("theoretical") relative frequency

$$p_i(nk) = \int_{i\Delta y}^{(i+1)\Delta y} \varphi(y;nk)\,\mathrm{d}y\,,\quad i = 1,\ldots,N_i\,. \tag{36}$$

RESULTS AND DISCUSSION

A time evolution of the dimensionless probability density $\varphi(y;t)$ resulting from numerical solution of Kolmogorov equation is depicted in Fig. 2. It is obvious that segregation of particles is considerably fast process. Already at $t = 0.0086$ (2 min of mixing) the concentration profile of greater particles along the mixer is well developed. With mixing time increasing above $t = 0.0173$ (4 min) the changes of $\varphi(y;t)$ become less meaningful. Good agreement of computed $\varphi(y;t)$ and experimental data is evident in Fig. 2. It means that simple method of diffusion coefficient evaluation based on application of Eq. *(10)* or *(12)* is quite correct in principle. Somewhat higher deviations between computed probability density and experimental data can be observed at the boundaries of the mixer, especially at the lowest value of the mixing time. The reason of these deviations arise probably from not completely perfect fulfilment of the assumption of elastic particle reflection on the mixer side walls (boundary condition *(15)*) and perhaps from certain dependence of diffusion coefficient on mixing time. A speculation on this possible dependence, however, comes out of frame of simple phenomenological model of segregation process presented in this paper.

Primary problem in numerical solution of a stochastic differential equation is a choice of the magnitude of time step (parameter k in Eq. *(33)*) and number of modelling particles (N_p in Eq. *(31)*). However, there is no straightforward method enabling a priori estimation of both parameters. The solution of Eq. *(32)* can be expressed either by the

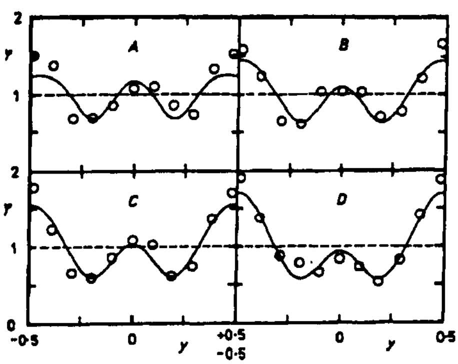

FIG. 2
Evolution of dimensionless transitive probability density in course of segregation process. — Kolmogorov equation solution, O experimental data. *A* $t = 0.0086$, *B* $t = 0.0173$, *C* $t = 0.0259$, *D* $t = 0.0432$

set of N_p + 1 random trajectories of individual particles in the mixer (see example in Fig. 3), or by probability distribution of particle positions at certain time (cf. Eq. (*35*)). Only the second form of the solution can be used for comparison of numerical simulation results to the measured particle concentrations and to the solution of corresponding Kolmogorov equation.

We have used chi-square criterion for testing of goodness of fit of the probability distribution resulting from numerical SDE solution and the dimensionless probability density $\varphi(y;t)$ resulting from solution of Eq. (*13*). The $\varphi(y;t)$ was converted to relative frequencies p_i using Eq. (*36*).

The values of k in the range 0.01 – 0.00001 and N_p = 3 000 were used for the solution of Eq. (*32*) and results at t = 0.4 (stationary state) were compared to the solution of Eq. (*13*) by means of chi-square test (N_i = 40). However, the computed values of χ^2 were lower than critical value (χ^2 = 54.6 at probability level 0.05) in all cases. It means, that the effect of time step magnitude on the solution of SDE is not recognisable by means of χ^2 test. We used time step k = 0.0001 in all simulations reported below, which was the same as used in numerical solution of Kolgomorov equation. The effect of N_p on solution of SDE is discussed in further paragraphs.

It is evident from Fig. 4 that the agreement of the SDE and Kolgomorov equation solutions expressed by particle relative frequencies is very good. The only small deviations are of random nature and correspond to stochastic property of the SDE solution. At all values of t in Fig. 4 the χ^2 criterion value was lower than the critical one. Therefore no significant difference between both equations results can be concluded.

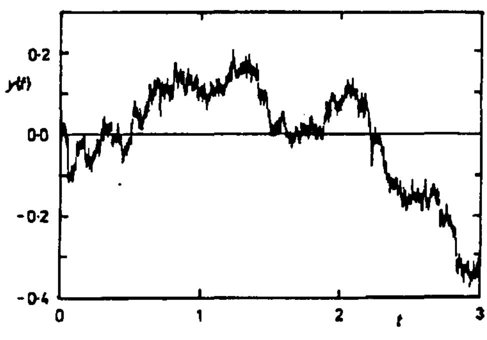

FIG. 3

Random trajectory of one particle in the mixer generated by numerical solution of Eq. (*34*)

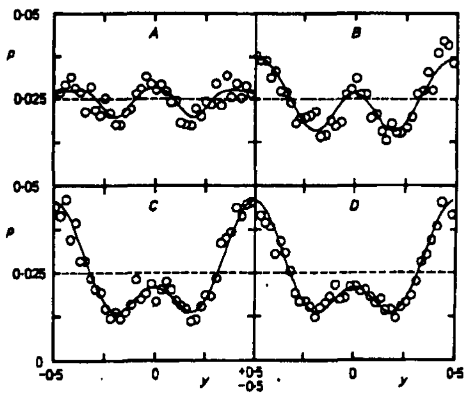

FIG. 4

Comparison of the SDE and Kolmogorov equation solutions (N_p = 3 000, N_i = 40). — Kolmogorov equation solution, O SDE solution. *A* t = 0.003, *B* t = 0.020, *C* t = 0.100, *D* t = 0.140

According to Chebyshev inequality (e.g. ref.[14]) the solution of the SDE should converge to the solution of Kolmogorov equation with increasing value of square root of N_p. There are two ways to increasing N_p value: (i) using higher N_p value in one simulation run (the maximum possible value is, however, restricted by capacity of a computer memory), or (ii) by repeating simulation with lower value of N_p and averaging of the resulting distributions. Both procedures were tested.

The effect of increasing number N_p in one simulation run on the SDE solution is demonstrated in Fig. 5. It is obvious from visual observation that the SDE solution with N_p = 30 000 is at all values of t somewhat more close to the Kolmogorov equation solution, than the SDE solution with N_p = 3 000 is. The χ^2 test (at time t = 0.140) resulted to these values: χ^2 = 46.1 (N_p = 3 000) and χ^2 = 245.0 (N_p = 30 000). Only the first value of χ^2 is lower than the critical value 54.6. Therefore the expected convergence of the SDE solution to Kolmogorov equation solution with increasing value of N_p was not proved by decreasing value of χ^2 criterion. The evaluation of the sums of squares of deviations gave these results: S_{sq} = 1.72 . 10^{-4} (N_p = 3 000) and S_{sq} = 1.55 . 10^{-4} (N_p = 30 000), i.e. the increase of N_p led to slightly improved agreement of the SDE solution and Kolmogorov equation solution. We can conclude that the χ^2 criterion is not suitable for testing of goodness of fit in the case of numerical simulation of the SDEs. The reason is, probably, that the value of χ^2 criterion is directly proportional to N_p value, whereas Chebyshev inequality predicts convergence proportional to the square root of N_p.

The effect of averaging of probability distributions resulting from the repeated numerical SDE simulations is documented in Fig. 6. The differences between the averaged

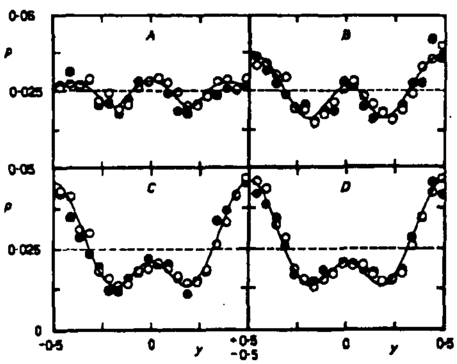

FIG. 5

The effect of number of modelling particles N_p on the SDE solution. ● N_p = 3 000, ○ N_p = 30 000, — Kolgomorov equation solution. *A* t = 0.003, *B* t = 0.020, *C* t = 0.100, *D* t = 0.140

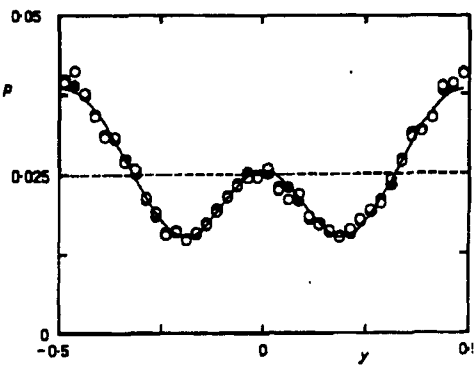

FIG. 6

The effect of averaging on the SDE solution, N_p = 3 000, t = 0.025. ○ 10 repeated simulations, ● 20 repeated simulations, — Kolmogorov equation solution

SDE solution both for 10 both for 20 repeated simulations and Kolmogorov equation solution are almost negligible. The values of χ^2 criterion and sum of squares of deviations are: $\chi^2 = 31.3$, $S_{sq} = 5.35 \cdot 10^{-5}$ for $N_r = 10$, and $\chi^2 = 20.2$, $S_{sq} = 2.90 \cdot 10^{-5}$ for $N_r = 20$. Both values of χ^2 are lower than the critical one, the values of sums of squares of deviations are by one order lower than those in previous paragraph. Therefore averaging of results of the repeated SDE simulations is much more effective from the point of view of convergence of the SDE solution to the Kolmogorov equation solution than use of high number of modelling particles in a single simulation run. A reason of this phenomenon may consist[15] in certain imperfectness of the random number generator when producing very long sequences of random numbers ($1.2 \cdot 10^8$ numbers at N_p = 30 000 and 4 000 simulation steps). The sequences of random numbers G generated by the method described in one of the previous paragraphs were checked for normality, randomness and periodicity, however, the length of tested sequences was only $10^4 - 10^5$ numbers due to the limited memory capacity of the computer used. All these "short" tests proved the generator to be correct.

CONCLUSIONS

The results of numerical computations previewed in preceding section proved that the stochastic model of the particle segregation process in the rotating horizontal drum mixer presented in this paper describes adequately the dynamics of the process, i.e. development of particle concentration profile with mixing time. The agreement of model simulation results with experimental data is very good. It was proved that results of direct numerical simulation of segregation process using stochastic differential equation are identical to the solution of corresponding Kolmogorov diffusion equation. The algorithm of numerical solution of stochastic differential equation is very simple, general and effective. It could be more attractive in a case of much more complicated model (multivariable processes) resulting to systems of stochastic differential equations when solution of system of Kolmogorov equations is very difficult, if not impossible at all.

SYMBOLS

b	constant in Eq. (*10*), m s^{-1}
B	constant in Eq. (*4*)
D	diffusion coefficient, m^2 s^{-1}
f	transitive probability density, m^{-1}
F^{-1}	inverse distribution function
G	random variable from Gaussian distribution
h	dimensionless axial coordinate step
H	dimensionless diffusion coefficient
j	index of particle
k	dimensionless time step
L	length of mixer, m

L'	half-length of mixer, m
M	uniform random variable
n	number of time steps
N_i	number of classes
N_p	number of particles
N_r	number of repeated simulations
$N(0,1)$	Gaussian distribution with zero mean and unity variance
P	probability
p	relative frequency
S_{sq}	sum of squares of deviations
t	dimensionless time
U	dimensionless Wiener process
u	integration variable in Eqs (*19*) and (*30*)
V	drift velocity, m s^{-1}
w	value of Wiener process, $s^{1/2}$
W	Wiener process, $s^{1/2}$
x	concentration of particles
X	random function of time
y	dimensionless axial coordinate
Y	dimensionless random function of time (particle position)
z	axial coordinate, m
Z	random function of time (particle position), m
α	time interval, s
γ	dimensionless time increment
λ	time interval, s
φ	dimensionless transitive probability density
Φ	distribution function
θ	time, s
τ	time, s
Subscripts	
0	related to origin, initial value
∞	related to stationary state
i	*i*-th time step, *i*-th class
j	*j*-th particle

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